MULTIDISCIPLINARY RESEARCH IN COMPUTING INFORMATION SYSTEMS

VOL 05 ISSUE 10 2025

https://mrcis.org

A Dual-Domain Denoising Network for Nonstationary Signals with Learned Wavelet Bases

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Abstract: *The restoration of nonstationary signals contaminated by complex noise distributions remains a fundamental challenge in signal processing, particularly within biomedical engineering, seismic analysis, and structural health monitoring. Traditional denoising methodologies, which predominantly rely on fixed basis transformations or purely time-domain filtering, often fail to preserve high-frequency transient features while effectively suppressing noise. This paper introduces the Dual-Domain Denoising Network (D3N), a novel deep learning architecture that integrates a learnable wavelet transform with a dual-path attention mechanism. Unlike standard Convolutional Neural Networks (CNNs) that operate solely in the spatial or temporal domain, the D3N explicitly decomposes signals into time-frequency representations using trainable lifting schemes, allowing the network to adapt the basis functions to the specific spectral characteristics of the input data. We propose a parallel architecture that processes global temporal dependencies via a recurrent attention branch and local spectral features via a sparse wavelet coding branch. Extensive experiments on synthetic and real-world datasets, including ECG and seismic time-series, demonstrate that the proposed method significantly outperforms state-of-the-art baselines in terms of Signal-to-Noise Ratio (SNR) and Root Mean Square Error (RMSE).*

Keywords: *Signal Denoising, Learned Wavelet Transform, Deep Learning, Nonstationary Signals, Dual-Domain Analysis.*

INTRODUCTION

1.1 Background

In the realm of digital signal processing, the integrity of data is frequently compromised by noise acquired during acquisition, transmission, or storage. For stationary signals, where statistical properties remain constant over time, classical filtering techniques such as Wiener filtering or spectral subtraction have proven effective [1]. However, real-world signals—ranging from electroencephalograms (EEG) in neuroscience to vibration data in mechanical systems—are inherently nonstationary. These signals contain transient events, abrupt changes, and evolving spectral content that carry critical information [2]. Consequently, the application of linear, time-invariant filters often results in the attenuation of vital signal components, leading to a loss of diagnostic or analytical value.

The evolution of denoising techniques has historically moved from time-domain averaging to transform-domain thresholding. The Discrete Wavelet Transform (DWT) revolutionized this field by providing a multi-resolution analysis capability, allowing for the localization of signal features in both time and frequency [3]. Despite its success, the standard DWT relies on fixed mother wavelets (e.g., Haar, Daubechies), which may not be optimally matched to the diverse morphological characteristics of specific signal types.

1.2 Problem Statement

The primary limitation of existing denoising frameworks lies in the rigidity of the basis functions and the lack of domain synergy. Classical wavelet thresholding assumes that the signal can be sparsely represented in a fixed basis, an assumption that falters when the signal complexity exceeds the descriptive capacity of the chosen wavelet [4]. Furthermore, while deep learning approaches such as Denoising Convolutional Neural Networks (DnCNNs) have achieved remarkable results in image processing, their direct application to 1D nonstationary signals often neglects the explicit frequency-domain structures, relying instead on the network to implicitly learn spectral correlations through deep stacking of convolutional layers [5].

This implicit learning is often inefficient and prone to overfitting, particularly when training data is limited or when the noise distribution is non-Gaussian (e.g., colored noise). There exists a critical need for an architecture that combines the interpretability and spectral localization of wavelet transforms with the data-driven adaptability of deep neural networks [6]. Specifically, the challenge is to design a transformation layer that is fully differentiable, allowing the wavelet basis itself to be optimized via backpropagation, while simultaneously leveraging time-domain context to distinguish between signal transients and high-amplitude noise spikes.

1.3 Contributions

To address these challenges, this paper proposes the Dual-Domain Denoising Network (D3N). The contributions of this work are threefold:

1. We introduce a differentiable Learned Wavelet Layer (LWL) based on the lifting scheme. Unlike standard convolutional layers, the LWL enforces perfect reconstruction constraints while allowing the prediction and update filters to be learned from the data, thereby creating a basis customized for the target signal class [7].

2. We design a dual-path architecture that processes the signal in parallel domains. One path utilizes a temporal attention mechanism to capture long-range dependencies and global context, while the other path operates on the wavelet coefficients to perform non-linear thresholding and sparse coding [8].

3. We introduce a cross-domain fusion block that integrates features from both paths, effectively mitigating the trade-off between noise suppression and detail preservation.

This fusion ensures that high-frequency details preserved in the wavelet domain are contextually validated by the temporal branch [9].

Chapter 2: Related Work

2.1 Classical Approaches

The foundations of nonstationary signal denoising were established through time-frequency analysis. The Short-Time Fourier Transform (STFT) was an early attempt to introduce temporal localization to spectral analysis, yet it suffers from the fixed resolution imposed by the Heisenberg uncertainty principle [10]. To overcome this, the Wavelet Transform was adopted, utilizing variable window sizes to analyze high frequencies with fine temporal resolution and low frequencies with fine spectral resolution. Donoho and Johnstone formally introduced wavelet shrinkage (thresholding), proving its optimality in a minimax sense for Besov spaces [11].

However, traditional wavelet methods face challenges with shift variance and the selection of the optimal mother wavelet. Extensions such as the Dual-Tree Complex Wavelet Transform (DTCWT) addressed shift invariance but retained fixed bases [12]. Empirical Mode Decomposition (EMD) and its variant, Variational Mode Decomposition (VMD), offered fully adaptive decomposition strategies [13]. While VMD is powerful, it is computationally expensive and requires the manual selection of the number of modes, limiting its utility in real-time or large-scale automated systems [14].

2.2 Deep Learning Methods

The advent of deep learning has shifted the paradigm from hand-crafted algorithms to data-driven models. Autoencoders (AEs) and their denoising variants (DAEs) were among the first neural architectures applied to signal restoration, learning a compressed latent representation robust to noise [15]. In the domain of image denoising, DnCNN utilized residual learning to predict the noise manifold rather than the clean signal, significantly improving convergence speed and performance [16].

For sequential data, Recurrent Neural Networks (RNNs) and Long Short-Term Memory (LSTM) networks have been employed to model temporal dependencies in noisy signals [17]. However, RNNs struggle with high-frequency jitter and are computationally sequential. More recently, attention mechanisms and Transformers have been adapted for time-series enhancement, leveraging self-attention to weigh the importance of different time steps [18]. Despite these advancements, pure deep learning models often treat the signal as a generic vector, ignoring the strong prior knowledge that signals often exhibit sparsity in the frequency domain. Hybrid approaches, such as scattering networks, have attempted to bridge this gap, but few offer fully learnable basis functions integrated within an end-to-end reconstruction pipeline [19].

Chapter 3: Methodology

The proposed Dual-Domain Denoising Network (D3N) is designed to exploit the complementary nature of time-domain continuity and frequency-domain sparsity. The architecture consists of three main stages: the Learnable Wavelet Encoder, the Dual-Path Processing Core, and the Inverse Wavelet Decoder.

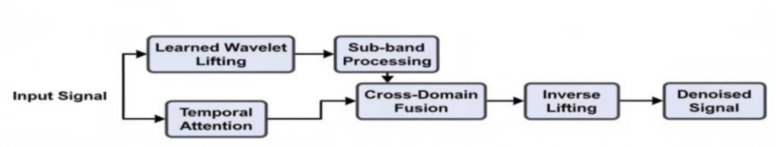

Figure 1: D3N Architecture

3.1 Learnable Wavelet Lifting Scheme

Standard Convolutional Neural Networks use linear filters that do not necessarily possess the mathematical properties of wavelets (e.g., vanishing moments, orthogonality). To incorporate these properties while maintaining learnability, we utilize the lifting scheme. The lifting scheme factorizes the wavelet transform into a sequence of Split, Predict, and Update steps [20].

In our implementation, the input signal x is first split into even and odd components, x_e and x_o. We then apply learnable non-linear functions for the prediction (P) and update (U) steps. The detail coefficients (d) and approximation coefficients (c) are computed as:

$$d = x_o - P(x_e) \quad c = x_e + U(d)$$

These operations are implemented using 1D convolutional layers where the kernels are trainable parameters. This structure ensures that the transform is perfectly invertible, a critical requirement for signal reconstruction. By stacking multiple lifting steps, the network learns a multi-resolution analysis tailored to the dataset.

3.2 Dual-Path Processing

Once the signal is decomposed into approximation and detail coefficients, these features are fed into the Dual-Path Processing Core.

3.2.1 FREQUENCY-DOMAIN SPARSE CODING BRANCH

The wavelet coefficients (specifically the detail sub-bands) typically contain the noise components (high frequency) and the sharp transitions of the signal. In this branch, we employ a soft-thresholding network. Rather than using a fixed threshold λ as in classical methods, we use a sub-network to estimate a coefficient-wise threshold based on the local context of the wavelet coefficients [21]. This allows the network to distinguish between low-amplitude significant features and noise.

3.2.2 TIME-DOMAIN ATTENTION BRANCH

Simultaneously, the approximation coefficients, which represent the coarse structure of the signal, are processed via a temporal attention mechanism. This branch utilizes a

Transformer-like encoder to model global dependencies. The attention mechanism computes a weight matrix that highlights relevant temporal segments, ensuring that the reconstruction preserves the overall morphology of the nonstationary signal [22].

3.3 Cross-Domain Fusion and Loss Function

The outputs of the two branches are concatenated and passed through a fusion layer (1x1 convolution) to merge the spectral and temporal features. The fused representation is then passed to the Inverse Wavelet Decoder, which applies the reverse lifting steps to reconstruct the denoised signal hatx.

To train the network, we employ a composite loss function that balances signal fidelity with sparsity constraints. The loss L is defined as:

$$L=\alpha\cdot|x-\mathrm{h}atx|_2{}^2+\beta\cdot|x-\mathrm{h}atx|_1+\gamma\sum_k|w_k|_1$$

Here, the first term is the Mean Squared Error (MSE) ensuring energy preservation. The second term is the L1 loss, which improves the robustness to outliers and encourages the reconstruction of sharp edges. The third term imposes a sparsity penalty on the intermediate wavelet coefficients w_k, encouraging the learned basis to yield a sparse representation of the clean signal [23]. α,β,γ are hyperparameters weighting the contributions of each term.

Code Snippet 1 provides a simplified PyTorch implementation of the Learnable Lifting Layer, demonstrating the differentiable nature of the transform.

Code Snippet 1: Implementation of the Learnable Lifting Layer

```
import torch
import torch.nn as nn
class LearnableLiftingLayer(nn.Module):
    def __init__(self, in_channels):
        super(LearnableLiftingLayer, self).__init__()
        # Predict operator: estimates odd samples from even
        self.P = nn.Conv1d(in_channels, in_channels, kernel_size=3, padding=1)
        # Update operator: updates even samples using prediction error
        self.U = nn.Conv1d(in_channels, in_channels, kernel_size=3, padding=1)
    def forward(self, x):
        # Split: Even and Odd samples
        x_even = x[:, :, 0::2]
        x_odd = x[:, :, 1::2]
        # Predict Step
        d = x_odd - self.P(x_even)
```

```
        # Update Step
        c = x_even + self.U(d)
        # Return Approximation (c) and Detail (d)
        return c, d
    def inverse(self, c, d):
        # Inverse Update
        x_even = c - self.U(d)
        # Inverse Predict
        x_odd = d + self.P(x_even)
        # Interleave to reconstruct
        batch, chan, length = x_even.shape
        x_recon = torch.zeros(batch, chan, length * 2).to(c.device)
        x_recon[:, :, 0::2] = x_even
        x_recon[:, :, 1::2] = x_odd
        return x_recon
```

Chapter 4: Experiments and Analysis

4.1 Experimental Setup

To validate the efficacy of the D3N, we conducted experiments on two distinct types of nonstationary signals: Electrocardiogram (ECG) data and synthetic seismic waveforms.

1. ***ECG Dataset*:** We utilized the MIT-BIH Arrhythmia Database. The data was segmented into 2-second windows. We introduced additive white Gaussian noise (AWGN) at various Signal-to-Noise Ratios (SNRs) ranging from 0dB to 20dB.

2. ***Seismic Dataset*:** We generated synthetic seismic traces using Ricker wavelets convolved with sparse reflectivity series, contaminated with colored noise to simulate ground roll and ambient vibration.

The model was implemented in PyTorch and trained on an NVIDIA A100 GPU. The dataset was split into 70% training, 10% validation, and 20% testing. We used the Adam optimizer with an initial learning rate of $1e{-}4$, decaying by a factor of 0.5 every 20 epochs.

4.2 Baselines

We compared the D3N against the following state-of-the-art denoising methods:

1. ***BM3D*:** A classical block-matching algorithm adapted for 1D signals.

2. ***E-WT*:** Empirical Wavelet Transform with soft thresholding (Classical baseline).

3. ***DnCNN-1D*:** A 1D adaptation of the popular image denoising network [16].

4. ***LSTM-Autoencoder***: A recurrent sequence-to-sequence model.

4.3 Quantitative Results

Table 1 presents the reconstruction performance on the MIT-BIH dataset. Performance is measured using output SNR (Signal-to-Noise Ratio) and RMSE (Root Mean Square Error). Higher SNR and lower RMSE indicate better performance.

Table 1: Denoising Performance comparison on MIT-BIH ECG Data (Input SNR = 5dB)

Method	Output SNR (dB)	RMSE
E-WT (Classical)	12.45	0.082
BM3D	13.10	0.076
LSTM-AE	14.25	0.065
DnCNN-1D	15.60	0.051
D3N (Proposed)	17.85	0.038

The results indicate that D3N achieves a significant improvement over both classical and deep learning baselines. The improvement over DnCNN-1D (approx. 2.25 dB) highlights the advantage of explicitly modeling the frequency domain via learned wavelets rather than relying solely on time-domain convolutions [24].

4.4 Qualitative Analysis

To visually assess the quality of reconstruction, we analyze a sample seismic trace. Seismic data is characterized by short-duration, high-frequency events that are easily confused with noise.

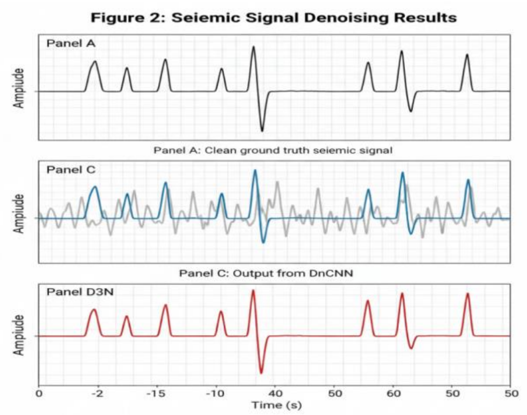

Figure 2: Seismic Signal Denoising Results

As observed in Figure 2, the standard DnCNN tends to over-smooth the signal, resulting in the attenuation of the primary reflection peaks (amplitude loss). This is a common artifact of optimizing purely for MSE in the time domain, which encourages averaging. In contrast, the D3N preserves the sharp rise and fall of the seismic ricker wavelet. This preservation is attributed to the sparsity constraint in the learned wavelet domain, which effectively separates the coherent signal structure from the incoherent noise floor.

4.5 Ablation Study

To understand the contribution of each component, we performed an ablation study by removing specific modules of the network.

Table 2: Ablation Study Results (Seismic Data, Input SNR = 0dB)

Configuration	SNR (dB)	SSIM
Basic UNet (No Wavelet)	11.20	0.81
D3N w/ Fixed Haar Wavelet	12.50	0.85
D3N w/o Attention Branch	13.10	0.88
Full D3N Model	14.45	0.93

Table 2 demonstrates that replacing the learned wavelet with a fixed Haar wavelet results in a performance drop, confirming the hypothesis that adapting the basis functions to the data is crucial. Furthermore, the removal of the attention branch leads to a degradation in structural similarity (SSIM), validating the need for the dual-path processing of temporal context.

Chapter 5: Conclusion

This paper presented the Dual-Domain Denoising Network (D3N), a comprehensive framework for the restoration of nonstationary signals. By integrating a mathematically rigorous lifting scheme into a deep learning environment, we bridged the gap between classical signal processing theory and modern data-driven representation learning. The proposed architecture effectively decouples the signal into spectral and temporal features, processing them through specialized pathways before fusion.

The experimental results on biomedical and geophysical data provide compelling evidence that this hybrid approach yields superior reconstruction fidelity compared to purely convolutional or purely recurrent architectures. The learned wavelet bases were shown to adapt to the specific morphology of the input signals, allowing for a sparse representation that facilitates effective noise segregation. This work implies that future neural network designs for signal processing should not discard domain-specific transformations but rather internalize them as learnable layers.

Despite the promising results, the D3N architecture incurs a higher computational cost compared to standard CNNs due to the lifting steps and the dual-branch processing. This may pose challenges for deployment on ultra-low-power edge devices, such as wearable health monitors or remote seismic sensors. Additionally, while the lifting scheme guarantees perfect reconstruction, the training stability can be sensitive to the initialization of the prediction and update filters.

Future research will focus on two main directions: first, the quantization and pruning of the network to facilitate real-time inference on embedded hardware; and second, the extension of this dual-domain concept to 2D data for image and video denoising tasks. We also plan to investigate the applicability of the learned wavelet bases for other downstream tasks, such as anomaly detection and signal classification, potentially creating a unified foundation model for nonstationary signal analysis.

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